Tricky Situation in Maximum Power Transfer Theorem in Special Case of an Amplifier

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Abstract

The maximum power transfer theorem is a very useful tool in applied electronics and electronic engineering. It has wide range of applications in amplifier. Its concept is as follows:

When a load is connected across a voltage source or across output of an amplifier, particular amount of power is transferred to the load. The amount of power being transferred depends on the value of the load resistance (RL). Its value is always unique, for that particular source. To adjust the maximum transfer of power from source to the load, the value of the load resistance and the value of internal resistance (Ri) of the source must be equal.

Introduction

In electrical engineering, the maximum power transfer theorem states that, to obtain maximum external power from a source with a finite internal resistance, the resistance of the load must equal the resistance of the source as viewed from its output terminals.

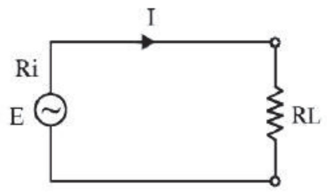

Fig:1.3 a

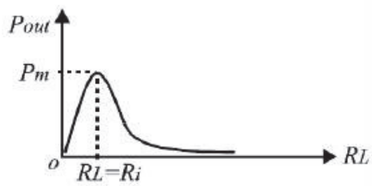

Fig:1.3 b

Suppose a voltage source *(E)* having internal resistance *(Ri)* and a load resistor *(RL)* are connected in parallel, as shown in *Fig:1.3a*. The current flowing through the circuit can be given as –

$$I = \frac{E}{R_L + R_i}$$

Power delivered to the load is given by –

$$P = I.R_L = \left(\frac{E}{R_L + R_i}\right)^2 .R_L \quad \dots\dots\dots (1)$$

Tricky Conditions

This theorem gives the impedance conditions in AC circuit for maximum power transfer to a load. It states that in an active AC network consisting of source with internal impedance Z_S which is connected to a load Z_L, the maximum power transfer occurs from source to load when the load impedance is equal to the complex conjugate of source impedance Z_S.

Consider the below circuit consisting of Thevenin's voltage source with series Thevenin's equivalent resistance *(which are actually replacing the complex part of the circuit)* connected across the complex load.

From the above figure, Let $Z_L = R_L + jX_L$ and $Z_{TH} = R_{TH} + jX_{TH}$ then the current through the circuit is given as:

$$I = \frac{V_{TH}}{Z_{TH} + Z_L}$$

By substituting for given impedance

$$I = \frac{V_{TH}}{R_{TH} + jX_{TH} + R_L + jX_L}$$

$$I = \frac{V_{TH}}{(R_L + R_{TH}) + j.(X_L + X_{TH})}$$

Power delivered to the load is $P_L = I^2.R_L$

$$P_L = \frac{(V_{TH})^2 \times R_L}{(R_L + R_{TH})^2 + (Z_L + Z_{TH})^2}$$

For power to be maximized, the above equation must be differentiated with respect to XL and equates it to zero.

$$X_L + X_{TH} = 0$$
$$X_L = -X_{TH}$$
$$P_L = \frac{(V_{TH})^2 \times R_L}{(R_L + R_{TH})^2}$$
$$R_L + R_{TH} = 2R_L$$

i.e. $R_L = R_{TH}$

Again taking derivative of the above equation and equating it to zero:

$$R_L + R_{TH} = 2R_L$$

Therefore, in AC circuits, if $X_L = -X_{TH}$ and $R_L = R_{TH}$, maximum power transfer takes place from source to load. This implies that maximum power transfer occurs when the impedance of the load is complex conjugate of the source impedance, i.e. $Z_L = Z_{TH}$

$$P_{max} = \frac{(V_{TH})^2}{4.R_{TH}} = \frac{(V_{TH})^2}{4.R_L}$$

Suppose we have a source having constant voltage and internal resistance. So the power delivered to the load will be directly proportional to the value of load resistance R_L. Now to find the value of R_L when maximum power is delivered to the load, we shall differentiate equation (1) w.r.t. R_L as follows –

$$\frac{dP}{dR_L} = E^2\left(\frac{(R_L + R_i)^2 - 2R_L.(R_L + R_i)}{(R_L + R_i)^4}\right) = 0$$

It means that –

$$(R_L + R_i)^2 - 2R_L.(R_L + R_i) = 0$$

Or

$$(R_L + R_i) + (R_L + R_i - 2R_L) = 0$$

Or

$$(R_L + R_i).(R_i - R_L) = 0$$

Since $(R_L + R_i) \neq 0$ therefore $(R_i - R_L)$ must be equal to zero.

Thus finally we can show that –

$$R_L = R_i$$

It means that load resistance must be equal to the internal resistance of the source.

Thus in Maximum Power Transfer Theorem, the load resistance R_L must match with the internal resistance of the source R_i. The above figure shows this condition in details. When $R_L = R_i$ the power delivered to the load will be equal to P_{max} .

The Tricky Situation

When we try to adjust the condition of Maximum Power Transfer Theorem in an amplifier, a very tricky situation arises.

For example, consider an amplifier system, in which we want to deliver maximum power output to the load connected at the output of the amplifier.

So when we try to satisfy the condition $R_L = R_i$, the efficiency of the amplifier system is sacrificed at the cost of delivering maximum power to the load.

Mathematically this can be explained as follows –

$$efficiency = \frac{output\ power}{input\ power} = \frac{I^2.R_L}{I^2.(R_L + R_i)} = \frac{R_L}{2.R_L} = \frac{1}{2} = 50\% \qquad becasue\ \ R_L = R_i$$

Conclusion

Hence we cannot always stick to satisfy the condition of Maximum Power Transfer Theorem. However this is applicable only when -

$$P_{max} = \frac{(V_{TH})^2}{4.R_{TH}} = \frac{(V_{TH})^2}{4.R_L}$$

References

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2) Baudrand, Henri. (1970). On the Generalizations of the Maximum Power Transfer Theorem. Proceedings of the IEEE. 58. 1780 - 1781. 10.1109/PROC.1970.8011