Kaluza-Klein Universe in $f(R, T)$ Gravity with Constant Deceleration Parameter in Presence of Quark Matter

Samadhan L. Munde

Department of Mathematics, Shri R. L. T. College of Science, Akola – 444001, Maharashtra, India
E-mail: *samadhan.munde86[at]gmail.com*

Abstract: ***In the present paper considered Kaluza-Klein cosmological model with quark matter and strange quark matter in f(R,T) theory of gravity which proposed by Harko et al. (2011). The general solutions of the field equations of Kaluza-Klein space-time have been obtained under the assumption of constant deceleration parameter in the context of power-law volumetric expansion model. The physical and geometrical aspects of the model are also discussed.***

Keywords: Kaluza-Klein Space-time, $f(R,T)$ theory of Gravity, Constant Deceleration Parameter

1. Introduction

A fundamental theoretical challenge to gravitational theories has been imposed by the observational data [1 – 6] on the late time acceleration of the universe and the existence of the dark matter. Carroll *et al*.[7] explained the presence of a late time cosmic acceleration of the universe in $f(R)$ gravity. Bertolami *et al*.[8] have proposed a generalization of $f(R)$ modified theories of gravity, by including in the theory an explicit coupling of an arbitrary function of the Ricci scalar R with the matter Lagrangian density L_m. Several $f(R)$ gravity models are reviewed by Capozziello & Faraoni [9]. Harko & Lobo [10] proposed a maximal extension of the Hilbert-Einstein action, by assuming that the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and of the matter Lagrangian L_m.

Harko *et al.* [11] developed $f(R,T)$ modified theory of gravity, in this theory the gravitational Lagrangian is given as an arbitrary function of the Ricci scalar R and also function of the stress-energy tensor T. They have obtained the gravitational field equations in the metric formalism, as well as, the equations of motion for test particles, which follow from the covariant divergence of the stress-energy tensor. Generally, the gravitational field equations depend on the nature of the matter source. They have presented the field equations of several particular models, corresponding to some explicit forms of the function $f(R,T)$. Reddy *et al*. [12, 13] have extended this work for Kaluza-Klein and Bianchi type-III Universe and Adhav [14] for LRS Bianchi type-I Universe in the presence of perfect fluid source in the framework of $f(R,T)$ gravity.

The theory of five dimensions is due to the idea of Kaluza [15] and Klein [16]. A five-dimensional general relativity is the best outcome of an attempt made by these two by using one extra dimension to unify gravity and electro-magnetism. Kaluza-Klein theory is essentially an extension of Einstein's general relativity in five dimensions which is of much interest in particle physics & cosmology.

The number of studies has been done by considering quark matter and strange quark matter in general relativity and other modified theories of gravity [17 – 20]. Recently, Prasad *et al.* [21] considered the bulk viscous fluid for the model in $f(R,T)$ gravity. Maurya *et al.* [22] have studied Domain walls and quark matter in Bianchi type-V universe with observational constraints in $f(R,T)$ gravity.

In the present paper, studied Kaluza-Klein cosmological model with Quark Matter (QM) and Strange Quark Matter (SQM) in $f(R,T)$ theory of gravity which proposed by Harko *et al.* [11]. The general solutions of the field equations of Kaluza-Klein space-time have been obtained under the assumption of constant deceleration parameter in the context of power-law volumetric expansion model. The physical and geometrical aspects of the model are also discussed in details.

2. Gravitational field equations in $f(R,T)$ theory of gravity

In $f(R,T)$ theory of gravity, the field equations are obtained from the Hilbert-Einstein type variation principle [11]. The action for this modified theory of gravity is given by

$$S = \frac{1}{16\pi}\int f(R,T)\sqrt{-g}\, d^4x + \int L_m\sqrt{-g}\, d^4x\,, \qquad (1)$$

where $f(R,T)$ is an arbitrary function of the Ricci scalar R and of the trace T of the stress-energy tensor of the matter $T_{\mu\nu}$ and L_m is the matter Lagrangian.

The corresponding field equations of the $f(R,T)$ gravity is found by varying the action (1) with respect to the metric $g_{\mu\nu}$:

$$f_R(R,T)R_{\mu\nu} - \frac{1}{2}f(R,T)g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)f_R(R,T) = 8\pi T_{\mu\nu} - f_T(R,T)T_{\mu\nu} - f_T(R,T)\Theta_{\mu\nu}\,, \qquad (2)$$

where $f_R(R,T) = \frac{\partial f(R,T)}{\partial R}\,,\ f_T(R,T) = \frac{\partial f(R,T)}{\partial T}\,,$

$$\Box = \nabla^\mu\nabla_\mu\,, \qquad \Theta_{\mu\nu} = g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{\mu\nu}}\,; \qquad (3)$$

∇_μ is the covariant derivative; $T_{\mu\nu}$ is the standard matter energy-momentum tensor derived from the Lagrangian L_m.

The stress-energy tensor of matter is

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta\,(\sqrt{-g}\,L_m)}{\delta g^{\mu\nu}} \tag{4}$$

The tensor $\Theta_{\mu\nu}$ in (2) is given by

$$\Theta_{\mu\nu} = -2\,T_{\mu\nu} + g_{\mu\nu}L_m - 2g^{\alpha\beta}\frac{\partial^2 L_m}{\partial g^{\mu\nu}\,\partial g^{\alpha\beta}} \tag{5}$$

the matter Lagrangian L_m may be chosen as $L_m = -p$, where p is the thermodynamical pressure of matter content of the Universe.

Now, equation (5) gives the variation of the stress-energy tensor as

$$\Theta_{\mu\nu} = -2\,T_{\mu\nu} - p\,g_{\mu\nu} \tag{6}$$

Generally, the field equations also depend on [through the tensor $\Theta_{\mu\nu}$] the physical nature of the matter field. Hence, several theoretical models corresponding to different matter sources in $f(R,T)$ gravity can be obtained. Harko *et al.*[11] obtained some particular classes of $f(R,T)$ modified gravity models by specifying functional form of f as

$$\left.\begin{array}{l} (i)\ \ f(R,T) = R + 2f(T) \\ (ii)\ \ f(R,T) = f_1(R) + f_2(T) \\ (iii)\ f(R,T) = f_1(R) + f_2(R)f_3(T) \end{array}\right\} \tag{7}$$

Harko *et al.* [11] have investigated FRW cosmological models in this theory by choosing appropriate function $f(T)$. They have also discussed the scalar fields play a vital role in cosmology. The equations of motion of test particles and a Brans-Dickey type formulation of the model are also presented.

3. Metric and Field Equations

Consider a five-dimensional Kaluza-Klein metric in the form as

$$ds^2 = dt^2 - A^2(t)(dx^2 + dy^2 + dz^2) - B^2(t)d\psi^2 \tag{8}$$

where $A(t)$ and $B(t)$ are the scale factors (metric tensors) and functions of cosmic time t only and the fifth coordinate ψ is taken to be space-like.

The energy momentum tensor for quark matter [23, 24] is

$$T^{(Quark)}{}_{\mu\nu} = (\rho + p)\,u_\mu u_\nu - pg_{\mu\nu} \tag{9}$$

where $\rho = \rho_q + B_c$ is the energy density, $p = p_q - B_c$ is pressure of the fluid and $u_\mu = (1,0,0,0,0)$ is the five-velocity vector in the comoving coordinates which satisfies the condition $u_\mu u^\mu = 1$. Since quark matter behaves nearly perfect fluid [23 – 27]and which is given in the following equation of state for quark matter as

$$p_q = \varepsilon\rho_q \quad , \quad 0 \le \varepsilon \le 1. \tag{10}$$

Also, the linear equation of state for strange quark matter [28 , 29] is given as

$$p = \varepsilon(\rho - \rho_0), \tag{11}$$

where ρ_0 is the energy density at zero pressure and ε is a constant.

When $\varepsilon = \frac{1}{3}$ and $\rho_0 = 4B_c$, the above linear equation of state is reduced to the following equation of state for strange quark matter in the bag model [23 , 24] as

$$p = \frac{(\rho - 4B_c)}{3}, \tag{12}$$

where B_c is the Bag constant.

In the present work, consider Kaluza-Klein cosmological model for the particular choice of $f(R,T)$ given by

$$f(R,T) = R + 2f(T)\,, \tag{13}$$

where the $f(T)$ is an arbitrary function of the trace of the stress-energy tensor of matter.

Using equations (6) and (13) in equation (2) then the gravitational field equation in $f(R,T)$ gravity becomes

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi T_{\mu\nu} + 2f'(T)T_{\mu\nu} + [2pf'(T) + fT g\mu\nu\,, \tag{14}$$

where prime denotes differentiation with respect to the argument.

Now, in the present work, we choose the function $f(T)$ of the trace of the stress-energy tensor of the matter as

$$f(T) = \lambda T, \text{ where } \lambda \text{ is a constant.} \tag{15}$$

The corresponding field equations (14) for metric (1) with the help of equations (9) & (15) can be written as

$$3\frac{\dot{A}^2}{A^2} + 3\frac{\dot{A}\dot{B}}{AB} = -(8\pi + 3\lambda)\rho_q + 2\lambda\,p_q - (8\pi + 5\lambda)B_c \tag{16}$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{B}}{B} = (8\pi + 4\lambda)p_q - \lambda\,\rho_q - (8\pi + 5\lambda)B_c \tag{17}$$

$$3\frac{\ddot{A}}{A} + 3\frac{\dot{A}^2}{A^2} = (8\pi + 4\lambda)p_q - \lambda\,\rho_q - (8\pi + 5\lambda)B_c\,, \tag{18}$$

where the overhead dot ($\dot{}$) denote derivative with respect to the cosmic time t.

The spatial volume (V) is defined as

$$V = a^4 = A^3B \tag{19}$$

Where a is the average scale factor.

The directional Hubble parameters in the directions of x, y, z and ψ axes respectively are defined as

$$H_x = H_y = H_z = \frac{\dot{A}}{A}\,, \quad H_\psi = \frac{\dot{B}}{B} \tag{20}$$

The mean Hubble parameter (H) is given by

$$H = \frac{1}{4}\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right). \tag{21}$$

The volumetric deceleration parameter (q) is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \tag{22}$$

The anisotropic parameter of the expansion (Δ) is defined as

$$\Delta = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right)^2, \tag{23}$$

where $H_i\,(i = 1,2,3,4)$ represent the directional Hubble parameters in the direction of x, y, z and ψ respectively.

The expansion scalar (θ) is defined as

$$\theta = 4H \,. \tag{24}$$

The Shear scalar(σ^2) is defined as

$$\sigma^2 = \frac{4}{2}\Delta\, H^2 \,. \tag{25}$$

4. Solutions of the field equations

In this section we find the solution of the field equations. Since there are three highly non-linear differential equations (16) to (18) with four unknowns$A$, B , ρ_q and p_q. In order to solve the system completely we impose a law of variation for the Hubble parameter which was initially proposed by Berman [30] for RW (Robertson-Walker) space-time and yields the constant value of deceleration parameter. According to this law, the variation of the mean Hubble parameter for the Kaluza-Klein metric (1) can be written as [31]

$$H = k(A^3B)^{-m/4} \,, \tag{26}$$

where$k > 0$ and $m \geq 0$are constants.

Equating equation (21) with (26) and integrating we get

$$V = A^3B = c_1 e^{4kt} \quad , \quad \text{for} = 0 \,, \tag{27}$$

and

$$V = A^3B = (mkt + c_2)^{4/m} \quad , \quad \text{for} \neq 0 \,, \tag{28}$$

where c_1 and c_2 are positive constants of integration.

Using equation (26) with (27) for $m = 0$ and with (28) for $\neq 0$, the mean Hubble parameters are

$$H = k \qquad , \qquad \text{for} = 0 \,, \tag{29}$$

and

$$H = k(mkt + c_2)^{-1} \quad , \quad \text{for} \neq 0 \,. \tag{30}$$

Using equations (27) and (28) in (22), one can obtain constant values for the deceleration parameter as

$$q = -1 \qquad , \qquad \text{for} = 0 \,, \tag{31}$$

and

$$q = m - 1 \quad , \qquad \text{for } m \neq 0. \tag{32}$$

The sign of q indicates whether the model accelerates or not. The positive sign if $q(m > 1)$ corresponds to decelerating models where as the negative sign $-1 \leq q < 0$ for $0 \leq m < 1$ indicates acceleration and $q = 0$for $m = 1$ corresponds to expansion with constant velocity.

In this paper, consider the model form $\neq 0 (q \neq -1)$:
(Pawer-Law Volumetric Expansion Model)

Subtracting equation (17) from (18) and using mean Hubble parameter from equation (30) and integrating, we get

$$\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = \frac{c_3}{(mkt + c_2)^{4/m}} \quad , \tag{33}$$

where c_3 is constant of integration.

On integration of (33) and using (28) we get exact expression for the scale factors:

$$A(t) = (c_4)^{1/4}\,(mkt + c_2)^{1/m}\; e^{\left\{\frac{c_3\,(mkt + c_2)^{\frac{m-4}{m}}}{4k(m-4)}\right\}}, \tag{34}$$

$$B(t) = (c_4)^{-3/4}\,(mkt + c_2)^{1/m}\; e^{\left\{\frac{-3\,c_3(mkt + c_2)^{\frac{m-4}{m}}}{4k(m-4)}\right\}}, \tag{35}$$

where c_4 is a constant of integration.

The spatial volume (V) of the universe is found to be

$$V = (mkt + c_2)^{4/m} \,. \tag{36}$$

The directional Hubble parameters in the directions of x, y, zandψ axes respectively are given by

$$H_x = H_y = H_z = \frac{k}{(mkt + c_2)} + \frac{c_3}{4}(mkt + c_2)^{-4/m} \,, \tag{37}$$

$$H_\psi = \frac{k}{(mkt + c_2)} - \frac{3\,c_3}{4}(mkt + c_2)^{-4/m} \tag{38}$$

The mean Hubble parameter (H) is found to be

$$H = \frac{k}{(mkt + c_2)} \,. \tag{39}$$

The anisotropic parameter (Δ) of the expansion is found to be

$$\Delta = \frac{3c_3^2}{16k^2}(mkt + c_2)^{2(m-4)/m} \,. \tag{40}$$

The expansion scalar (θ) is found to be

$$\theta = \frac{4k}{(mkt + c_2)} \tag{41}$$

The Shear scalar (σ^2) is found to be

$$\sigma^2 = \frac{3\,c_3^2}{8}(mkt + c_2)^{-8/m} \tag{42}$$

Using equations (34) and (35) in equation (16) with the help of linear equation of state (10) for $\varepsilon = \frac{1}{3}$, we can obtain the energy density and pressure for the quark matter as

$$\rho_q = \frac{9\,c_3^2(mkt + c_2)^{-8/m}}{8\,(24\pi + 7\lambda)} - \frac{18\,k^2(mkt + c_2)^{-2}}{(24\pi + 7\lambda)} - \frac{3(8\pi + 5\lambda)B_c}{(24\pi + 7\lambda)} \tag{43}$$

$$p_q = \frac{3\,c_3^2(mkt + c_2)^{-8/m}}{8\,(24\pi + 7\lambda)} - \frac{6\,k^2(mkt + c_2)^{-2}}{(24\pi + 7\lambda)} - \frac{(8\pi + 5\lambda)B_c}{(24\pi + 7\lambda)} \tag{44}$$

Similarly, using equations (34) and (35) in equation (16) with the help of linear equation of state (12), we obtain the energy density and pressure for the strange quark matter as

$$\rho = \frac{9\,c_3^2(mkt + c_2)^{-8/m}}{8\,(24\pi + 7\lambda)} - \frac{18\,k^2(mkt + c_2)^{-2}}{(24\pi + 7\lambda)} - \frac{(24\pi + 23\lambda)B_c}{(24\pi + 7\lambda)}. \tag{45}$$

$$p = \frac{3\,c_3^2(mkt + c_2)^{-8/m}}{8\,(24\pi + 7\lambda)} - \frac{6\,k^2(mkt + c_2)^{-2}}{(24\pi + 7\lambda)} - \frac{(40\pi + 17\lambda)B_c}{(24\pi + 7\lambda)}. \tag{46}$$

5. Discussion and Conclusion

In this paper, the Kaluza-Klein cosmological model with quark and strange quark matters in$f(R, T)$gravity has been studied. The solutions of the field equations have been obtained by using a law of variation for the Hubble parameter which yield the constant value of deceleration parameter.

Equations (34) and (35) gives the solution of Kaluza-Klein cosmological model for power-law volumetric expansion in $f(R, T)$gravity. From equations (34) and (35), it is observed that as$t \to 0$,

$$A(t) \to (c_4)^{1/4}(c_2)^{1/m}\,.\,exp\left[\,(c_3.c_2^{\frac{m-4}{m}})/4k(m-4)\right],$$

$$B(t) \to (c_4)^{-3/4}\,(mkt+c_2)^{1/m}\,.\,exp\left[(-3c_3.c_2^{\frac{m-4}{m}})/4k(m-4)\right]$$

and as $t \to \infty$, $A(t) \to \infty$, $B(t) \to \infty$.

From equations (37) and (38), it is observed that, the directional Hubble parameters H_x, H_y and H_z are finite at $t = 0$ and vanishes at $t = \infty$.

In power-law volumetric expansion model, it is also observed that the spatial volume V is finite at $t = 0$ and become infinitely large as $t \to \infty$ as shown in following figure-1. The anisotropy of the expansion is not promoted by the anisotropy of the fluid. Here the anisotropy of the expansion $\Delta \to$ constant at $t \to 0$ and then decreases to null as t increases provided that $0 < m < 4, k = c_2 = c_3 = 1$. The space approaches to isotropy in this model since $\Delta \to 0$ as $t \to \infty$ as shown in figure-2.

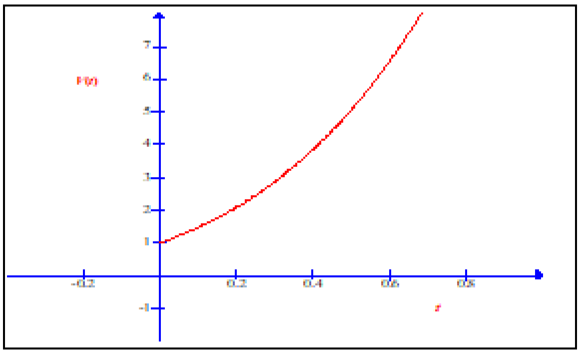

Figure 1: Variation of spatial volume $V(t)$ against cosmic time t for $m = k = c_2 = 1$.

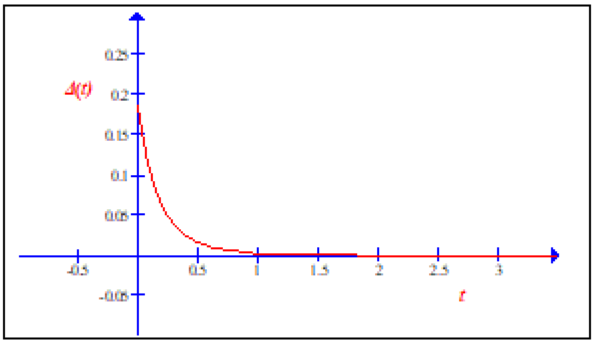

Figure 2: Variation of anisotropic parameter (Δ) against cosmic time t for $m = k = c_2 = c_3 = 1$.

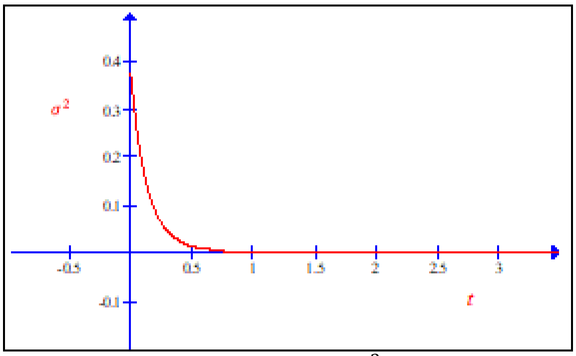

Figure 3: Variation of shear scalar (σ^2) against cosmic time t for $m = k = c_3 = 1$.

From equation (42), it is observed that the shear scalar (σ^2) start with finite value at $t = 0$ and as time increases it decreases then tends to zero at $t \to \infty$ as shown in figure-3. The ratio $\frac{\sigma^2}{\theta^2} = \left(\frac{3c_3^2}{128k^2}\right)\frac{1}{(mkt+c_2)^{(8-2m)/m}} \to 0$ as $t \to \infty$ only provided that $0 < m < 4$ and hence the model isotropizes for large value of t. From equations (43) to (46), the density and pressure of quark matter (including strange quark models) become constants when $t \to 0$ and then decreases as t increases and remain constant when $t \to \infty$.

For power-law volumetric expansion model, it is observed that $q < 0$ for $0 \leq m \leq 1$ which indicate that the universe is accelerating. In particular, from equation (32) for $m = 1$, we get $q = 0$ which indicating that the universe is expanding with constant velocity, thus the results are matches with the Riess *et al.*[1] and Perlmutter *et al.*[2] that the deceleration parameter of the universe is in the range $-1 \leq q \leq 0$ and the present-day universe is undergoing accelerated expansion.

References

[1] Riess, A.G., *et al.*: Astron. J. **116**, 1009 (1998).
[2] Perlmutter, S., *et al.*: Astrophys. J. **517**, 565 (1999).
[3] Bernardis *et al.*: Nature, **404**, 955 (2000).
[4] Hanany *et al.*, : Astrophys. J. **545**, L5 (2000).
[5] Peebles, P. J. E. and Ratra, B. : Rev. Mod. Phys. **75**, 559 (2003).
[6] Padmanabhan, T.: Phys. Repts. **380**, 235 (2003).
[7] Carroll, *et al.* : Phys. Rev. D. **70,** 043528 (2004).
[8] Bertolami, *et al.* : Phys. Rev. D **75**, 104016 (2007).
[9] Capozziello, S. and Faraoni, V. : Beyond Einstein Gravity, Springer, (2010).
[10] Harko, T. and Lobo,F.S.N. : Eur. Phys. J. **C 70**, 373 (2010).
[11] Harko, T. *et.al*.: Phys. Rev. D **84**, 024020 (2011).
[12] Reddy D. R. K., *et al.* : Astrophys and Space Sci. **342**, 249 (2012).
[13] Reddy D. R. K., *et al.* : Int J Theor Phys. **51**, 3222 (2012).
[14] Adhav K. S. : Astrophys. and Space Sci. **339**(2), 365 (2012).
[15] Kaluza T.: Zum Unitats Prob der Physik Sitz Press. Akad.Wiss.Phys.Math.K1, 966 (1921).
[16] Klein, O.: Zeits.Phys. **37**,895 (1926).
[17] Katore, S.D.: Int. J. Theor. Phys. **51**, 8389 (2012).
[18] Mahanta, K.L., *et al*.: Eur. Phys. J. Plus **129**, 1 (2014).
[19] Santhikumar, R.B.S., *et al.*: Int. J. Phys. Math. Sci. **5**, 40 (2015).
[20] Caglar, H., *et al.*: Chin. Phys. C **40**, 045103 (2016).
[21] Prasad, R.: Pramana J. Phys. , **94**, 135 (2020).
[22] Dinesh Chandra Maurya , *et.al.*: Int.Jour.of Geo.Meth. in Mod.Phys.,**17**(01), 2050014 (2020).
[23] Aktas C., *et al*.: Gen. Relat. Gravit. **43**, 1577 (2011).
[24] Yilmaz, I. , *et al*.: Gen. Relat. Gravit. **44**, 2313 (2012).
[25] Back, B.B., *et al.*: (PHOBOS Collaboration), Nucl. Phys. A **757**, 28 (2005).
[26] Adams, J., *et al*.: (STAR Collaboration), Nucl. Phys. A **757**, 102 (2005).
[27] Adcox, K., *et al.*: (PHENIX Collaboration), Nucl. Phys. A **757**, 184 (2005).
[28] Sharma R. *et al*.: Int.Jour.Mod.Phy.D **15**, 405 (2006).

Paper ID: SR21416192615 DOI: 10.21275/SR21416192615

[29] Sharma R. and Maharaj S.D. : Mon. Not. R. Astron. Soc., **375**, 1265 (2007).
[30] Berman, M.S.: Nuovo Cimento B **74**, 182 (1983).
[31] Adhav,K.S. *et al*.: Modern Physics Letters A, **26**(10),739 (2011).