Assignment:

① Using stokes theorem evaluate $\int_c [(2x-y)dx - yz^2 dy - y^2 z\,dz]$ where c is the circle $x^2+y^2=1$, corresponding to the surface of sphere of unit radius.

Solution: Given that,

$$\int_c [(2x-y)dx - yz^2 dy - y^2 z\,dz] \quad \text{which is } \int_c \vec{F}\cdot d\vec{r}$$

$$\therefore \int_c \vec{F}\cdot d\vec{r} = \int_c [(2x-y)dx - yz^2 dy - y^2 z\,dz]$$

$$= \int_c [(2x-y)\hat{i} - yz^2\hat{j} - y^2 z\hat{k}]\cdot[\hat{i}dx + \hat{j}dy + \hat{k}dz]$$

$$\because d\vec{r} = \hat{i}dx + \hat{j}dy + \hat{k}dz$$

Therefore,

$$\vec{F} = (2x-y)\hat{i} - yz^2\hat{j} - y^2 z\hat{k}$$

$\therefore$ According to Stokes theorem.

$$\oint \vec{F}\cdot d\vec{r} = \iint_s \text{curl}\,\vec{F}\cdot\hat{n}\,ds \quad \text{— ①}$$

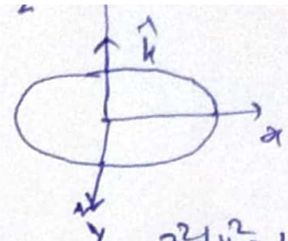
$$\therefore \text{curl}\,\vec{F} = \nabla\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2x-y & -yz^2 & -y^2 z \end{vmatrix}$$

$$= \hat{i}\left[\frac{\partial}{\partial y}(-y^2 z) - \frac{\partial}{\partial z}(-yz^2)\right]$$
$$-\hat{j}\left[\frac{\partial}{\partial x}(-y^2 z) - \frac{\partial}{\partial z}(2x-y)\right]$$
$$+\hat{k}\left[\frac{\partial}{\partial x}(-yz^2) - \frac{\partial}{\partial y}(2x-y)\right]$$
$$= \hat{i}[-2yz + 2yz] - \hat{j}[0-0]$$
$$+\hat{k}[0-(-1)]$$

$$\nabla\times\vec{F} = \hat{k}$$

putting this value i.e. $\nabla\times\vec{F} = \hat{k}$ in eq^n ① we get

$$\therefore \text{eq}^n\text{ ①} \Rightarrow \oint \vec{F}\cdot d\vec{r} = \iint_s \nabla\times\vec{F}\cdot\hat{n}\,ds$$



$x^2+y^2=1$

$\therefore \oint \bar{F}\cdot d\bar{r} = \iint \hat{k}\cdot\vec{n}\, ds$

$= \iint \hat{k}\cdot\hat{k}\, ds$

$= \iint ds$

$= \iint dx\,dy$

$= \int dx \int dy$

vector $\hat{k}$ is $\perp^{er}$ to ~~x~~-y plane

$\because \hat{n} = \frac{d\vec{s}}{ds}$

$d\vec{s} = \hat{n}\, ds$

$\because \hat{n} = \hat{k}$

$\therefore d\vec{s} = \hat{k}\, ds$

$\because$ Here, c is the circle,

$x^2+y^2 = 1$

$\therefore x^2 = 1$ Put, $y=0$

$\boxed{x = \pm 1}$ $\therefore x$ varies from -1 to $+1$

$\therefore y^2 = 1-x^2$

$\boxed{y = \pm\sqrt{1-x^2}}$ y varies from $-\sqrt{1-x^2}$ To $+\sqrt{1-x^2}$

$\therefore \oint \bar{F}\cdot d\bar{r} = \iint dx\,dy$

$= \int_{-1}^{+1} dx \int_{-\sqrt{1-x^2}}^{\sqrt{1+x^2}} dy$

$= \int_{-1}^{+1} dx\, [y]_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}$

$= \int_{-1}^{+1} dx\left[\sqrt{1-x^2} - (-\sqrt{1-x^2})\right]$

$= \int_{-1}^{+} dx\left[2\sqrt{1-x^2}\right]$

$= 2\int_{-1}^{1} \sqrt{1-x^2}\, dx$

$= 2\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}\left(\frac{x}{1}\right)\right]_{-1}^{+1} = 2\left[0 + \frac{1}{2}(\pi/2) - \frac{1}{2}(-\pi/2)\right]$

$= \pi$